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Chapter 5

Managing Interest Rate Risk: Duration Gap and Market Value of Equity

Duration and price volatility

- Maturity simply identifies how much time elapses until final payment.
- It ignores all information about the timing and magnitude of interim payments.
- Duration is a measure of effective maturity that incorporates the timing and size of a security's cash flows.
- Duration captures the combined impact of market rate, the size of interim payments and maturity on a security’s price volatility.

Duration versus maturity

1.) 1000 loan, principal + interest paid in 20 years.

2.) 1000 loan, 900 principal in 1 year,
100 principal in 20 years.

```
1                                       1000 + int
     |--------------------|-------------------|
     0                   10                  20
_________________________________________________

          900+int                    100 + int
2    |----|---------------|-------------------|
     0    1              10                  20
```

Duration
...approximate measure of the price elasticity of demand

- Price elasticity of demand = %Δ in quantity demanded / %Δ in price
- Price (value) changes
  - Longer duration → larger changes in price for a given change in i-rates.
  - Larger coupon → smaller change in price for a given change in i-rates.

Duration

...approximate measure of the price elasticity of demand

- Solve for ΔPrice:
  - $\Delta P \cong -\text{Duration} \times [\Delta i \,/\, (1 + i)] \times P$
  - $$\textbf{DUR} \cong -\left[\frac{\dfrac{\Delta P}{P}}{\dfrac{\Delta i}{1+i}}\right] \cong -\frac{\%\Delta\Delta}{\%\Delta i}$$
- Price (value) changes
  - Longer maturity/duration larger changes in price for a given change in i-rates.

Measuring duration

- In general notation, Macaulay's duration (D):

$$D = \frac{\sum_{t=1}^{k} \frac{CF_t(t)}{(1+r)^t}}{\sum_{t=1}^{k} \frac{CF_t}{(1+r)^t}} = \frac{\sum_{t=1}^{n} \frac{CF_t(t)}{(1+r)^t}}{\text{PV of the Sec.}}$$

Measuring duration

- Example: 1000 face value, 10% coupon, 3 year, 12% YTM

$$D = \frac{\frac{100 \times 1}{(1.12)^1} + \frac{100 \times 2}{(1.12)^2} + \frac{100 \times 3}{(1.12)^3} + \frac{1000 \times 3}{(1.12)^3}}{\sum_{t=1}^{3} \frac{100}{(1.12)^t} + \frac{1000}{(1.12)^3}} = \frac{2597.6}{951.96} = 2.73 \text{ years}$$

Measuring duration

- If YTM = 5%

  1000 face value, 10% coupon, 3 year, 5% YTM

$$D = \frac{\frac{100*1}{(1.05)^1} + \frac{100*2}{(1.05)^2} + \frac{100*3}{(1.05)^3} + \frac{1000*3}{(1.05)^3}}{1136.16}$$

$$D = \frac{3127.31}{1136.16} = 2.75 \text{ years}$$

Measuring duration

- If YTM = 20%

  1000 face value, 10% coupon, 3 year, 20% YTM

$$D = \frac{2131.95}{789.35} = 2.68 \text{ years}$$

Measuring duration

- If YTM = 12% and Coupon = 0

1000 face value, 0% coupon, 3 year, 12% YTM

```
                          1000
|--------|--------|--------|
0        1        2        3
```

$$D = \frac{\frac{1000 \times 3}{(1.12)^3}}{\frac{1000}{(1.12)^3}} = 3 \text{ (by definition}$$

Duration and modified duration

- Duration allows market participants to estimate the relative price volatility of different securities:

$$\frac{\Delta P}{P} \cong -\left[\frac{\text{Macaulay's Duration}}{1+y}\right]\Delta y$$

- Using modified duration:

  modified duration = Macaulay's duration / (1+y)

  - We have an estimate of price volatility:

    %change in price = modified duration x Δy

Effective Duration

- Effective duration is

$$\text{Effective duration} = \frac{P_{i-} - P_{i+}}{P_0(i^+ - i^-)}$$

  - Where
    P_{i-} = price if rates fall,
    P_{i+} = price if rates rise;
    P_0 = initial (current) price;
    i^+ initial market rate plus the increase in rate;
    i^- = initial market rate minus the decrease in rate.
- Effective duration compares a security's estimated price in a falling and rising rate environment.

Effective duration example

- Consider a 3-year, 9.4 percent coupon bond selling for $10,000 par to yield 9.4 percent to maturity.
- The callable bond's effective duration for a 30 basis point (0.30 percent) semiannual movement in rates either up or down is 2.54:

$$\text{Eff Dur} = \frac{\$10,000 - \$9,847.72}{\$10,000(0.05 - 0.044)} = 2.54$$

Two types of interest rate risk

- Reinvestment rate risk
  - Cost of funds versus the return on assets
    - ⇒ Funding GAP, impact on NII
- Price Risk
  - Longer maturity (duration)

 ⇒ larger change in value for a given change in interest rates

 ⇒ Duration GAP, impact on market value of equity

Reinvestment rate risk and price risk

- Reinvestment rate risk:
  - If i-rate ↑, then yield from reinvestment of the cash flows ↑ and holding period return (HPR) increases.
- Price risk:
  - If i-rate ↑ and you sell the security prior to maturity, the price or value falls, hence HPR falls.
- An *immunized security* is one in which the gain from the higher reinvestment rate is just offset by the capital loss.

There are four steps in duration gap analysis.

1. Management develops an interest rate forecast.
2. Management estimates the market value of bank assets, liabilities and stockholders' equity.
3. Management estimates the weighted duration of assets and weighted duration of liabilities.
4. Management forecasts changes in the market value of stockholders' equity across different interest rate environments.

Duration GAP at the bank

- The bank can protect either the market value of equity (MVE) or the book value of NII, but not both.
- To protect the MVE the bank would set DGAP to zero:
  - DGAP = DA - u x DL
  - where
    DA = weighted average duration of assets
    DL = weighted average duration of liabs

Focus on the market value of equity (MVE)

- We know that:
  $\Delta MVE = \Delta MVA - \Delta MVL$
  - With
    - $\Delta A_i = - DA_i \, [\Delta y / (1+y)] \, A_i$ and
    - $\Delta Lj = - DL_j \, [\Delta y / (1+y)] \, L_j$
- Hence:

$$\Delta MVE = -[DA - (MVL / MVA) \, DL] \, [\Delta y / (1+y)] \, MVA$$

- If we define a bank's duration gap:
  $(DGAP) = DA - (MVL / MVA) \, DL$, then
- $\Delta MVE = - DGAP \, [\Delta y / (1+y)] \, MVA$

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Hypothetical Bank Balance Sheet

	Par $1,000	% Coup	Years Mat.	YTM	Market Value	Dur.
Assets						
Cash	100				100	
Earning assets						
3-yr Comme					700	2.69
6-yr Treasu					200	4.99
Total Earni					900	
Non-cash ea					0	
Total assets					000	2.88
Liabilities						
Interest bearing liabs.						
1-yr Time deposit	620	5.00%	1	5.00%	620	1.00
3-yr Certificate of depos	300	7.00%	3	7.00%	300	2.81
Tot. Int Bearing Liabs.	920			5.65%	920	
Tot. non-int. bearing	0				0	
Total liabilities	920			5.65%	920	1.59
Total equity	80				80	
Total liabs & equity	1000				1000	

$$D=\frac{\frac{84\times1}{(1.12)^1}+\frac{84\times2}{(1.12)^2}+\frac{84\times3}{(1.12)^3}+\frac{700\times3}{(1.12)^3}}{700}$$

Calculating DGAP

- DA = (700/1000)*2.69 + (200/1000)*4.99 = 2.88
- DL = (620/920)*1.00 + (300/920)*2.81 = 1.59
  - DGAP = 2.88 - (920/1000)*1.59 = 1.42 years
- What does 1.42 mean?
  - The average duration of assets > liabilities,
    - hence asset values change by more than liability values.

1 percent increase in all rates.

	Par $1,000	% Coup	Years Mat.	YTM	Market Value	Dur.
Assets						
Cash	100				100	
Earning assets						
3-yr Commercial loan	700	12.00%	3	13.00%	683.47	2.69
6-yr Treasury bond	200	8.00%	6	9.00%	191.03	4.97
Total Earning Asset	[illegible]			[illegible]	874.5	
Non-cash earning as					0	
Total assets					974.5	2.86
Liabilities						
Interest bearing liabs.						
1-yr Time deposit	620	5.00%	1	6.00%	614.15	1.00
3-yr Certificate of depos	300	7.00%	3	8.00%	292.27	2.81
Tot. Int Bearing Liabs.	920			6.64%	906.42	
Tot. non-int. bearing	0				0	
Total liabilities	920			6.64%	906.42	1.58
Total equity	80				68.08	
Total liabs & equity	1000				974.5	

$$PV = \sum_{t=1}^{3} \frac{84}{1.13^t} + \frac{700}{1.13^3}$$

Calculating DGAP

- DA = (683 / 974) * 2.68 + (191 / 974) * 4.97
  = 2.86 yrs
- DA = (614 / 906) * 1.00 + (292 / 906) * 2.80
  = 1.58 yrs
  - DGAP = 2.86 - (906 / 974) * 1.58
    = 1.36 years
- What does 1.36 mean?
  The average duration of assets > liabilities, hence asset values change by more than liability values.

Change in the Market Value of Equity

- Using the relationship:

$$\text{DUR} \cong -\left[\frac{\frac{\Delta P}{P}}{\frac{\Delta y}{1+y}}\right] \cong -\frac{\%\Delta P}{\%\Delta y}$$

- We can define the change in the MVE as:

$$\Delta\text{MVE} \cong (-\text{DGAP}) \times \left[\frac{\Delta y}{(1+i_{\text{Total assets}})}\right] \times \text{TA}$$

- ΔMVE = (-1.42) x [+0.01 / (1.10)] x 1,000
  = -$12.90

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Positive and negative DGAPs

- Positive DGAP
  ...indicates that assets are more price sensitive than liabilities, on average.
  - Thus, when interest rates rise (fall), assets will fall proportionately more (less) in value than liabilities and the MVE will fall (rise) accordingly.
- Negative DGAP
  ...indicates that weighted liabilities are more price sensitive than assets.
  - Thus, when interest rates rise (fall), assets will fall proportionately less (more) in value that liabilities and the MVE will rise (fall).

An immunized portfolio

... What is the minimum risk position?

- To eliminate the risk of changes in the MVE, how much must DA or DL change by?
  - Change DA = -1.42
  - Change DL = +1.42/u = 1.54

Immunized portfolio

	Par $1,000	% Coup	Years Mat.	YTM	Market Value	Dur.
Assets						
Cash	100				100	
Earning assets						
3-yr Commercial loan	700	12.00%	3	12.00%	700	2.69
6-yr Treasury bond	200	8.00%	6	8.00%	200	4.99
Total Earning Assets	900			11.11%	900	
Non-cash earning assets	0				0	
Total assets	1000			10.00%	1000	2.88
Liabilities						
Interest bearing liabs.						
1-yr Time deposit	340	5.00%	1	5.00%	340	1.00
3-yr Certificate of depos	300	7.00%	3	7.00%	300	2.81
6-yr Zero-coupon CD*	444.3	0.00%	6	8.00%	280	6.00
Tot. Int Bearing Liabs.	1084.3			6.57%	920	
Tot. non-int. bearing	0				0	
Total liabilities	1084.3			6.57%	920	3.11
Total equity	80				80	

Immunized portfolio:
1% increase in all rates

	Par $1,000	% Coup	Years Mat.	YTM	Market Value	Dur.
Assets						
Cash	100				100	
Earning assets						
3-yr Commercial loan	700	12.00%	3	13.00%	683.47	2.69
6-yr Treasury bond	200	8.00%	6	9.00%	191.03	4.97
Total Earning Assets	900			12.13%	874.5	
Non-cash earning assets	0				0	
Total assets	1000			10.88%	974.5	2.86
Liabilities						
Interest bearing liabs.						
1-yr Time deposit	340	5.00%	1	6.00%	336.79	1.00
3-yr Certificate of depos	300	7.00%	3	8.00%	292.27	2.81
6-yr Zero-coupon CD*	444.3	0.00%	6	9.00%	264.94	6.00
Tot. Int Bearing Liabs.	1084.3			7.54%	894	
Tot. non-int. bearing	0				0	
Total liabilities	1084.3			7.54%	894	3.07
Total equity	80				80	

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Stabilizing the book value of net interest income

- This can be done for a 1-year time horizon, with the appropriate duration gap measure:

  DGAP* MVRSA(1- DRSA) - MVRSL(1- DRSL)

  If DGAP* is positive, the bank's net interest income will decrease when interest rates decrease, and increase when rates increase.
  - If DGAP* is negative, the relationship is reversed.
  - Only when DGAP* equals zero is interest rate risk eliminated.
- Banks can use duration analysis to stabilize a number of different variables reflecting bank performance.

Market value of equity sensitivity analysis

- MVE sensitivity analysis effectively involves the same steps as earnings sensitivity analysis.
- In MVA analysis, however, the bank focuses on:
  - the relative durations of assets and liabilities,
  - how much the durations change in different interest rate environments, and
  - what happens to the market value of equity across different rate environments.

Embedded options

- Prepayments that exceed (fall short of) that expected will shorten (lengthen) duration.
- A bond being called will shorten duration.
- A deposit that is withdrawn early will shorten duration.
  - A deposit that is not withdrawn as expected will lengthen duration.

Duration Gap for ABC's MVE

1. Mkt value of assets = 1,001,963
   Duration of assets = 2.6
2. Mkt value of liabilities = 919,400
   Duration of liabilities = 2.0

   Dur Gap= 2.6 – (919,400 / 1,001,963) * 2.0

   = 0.765 yrs

- Example:
  - A 1% increase in rates would reduce MVE by 7.2 million
  - = 0.765 (0.01 / 1.0693) 1,001,963

Recall that the average rate on assets is 6.93%

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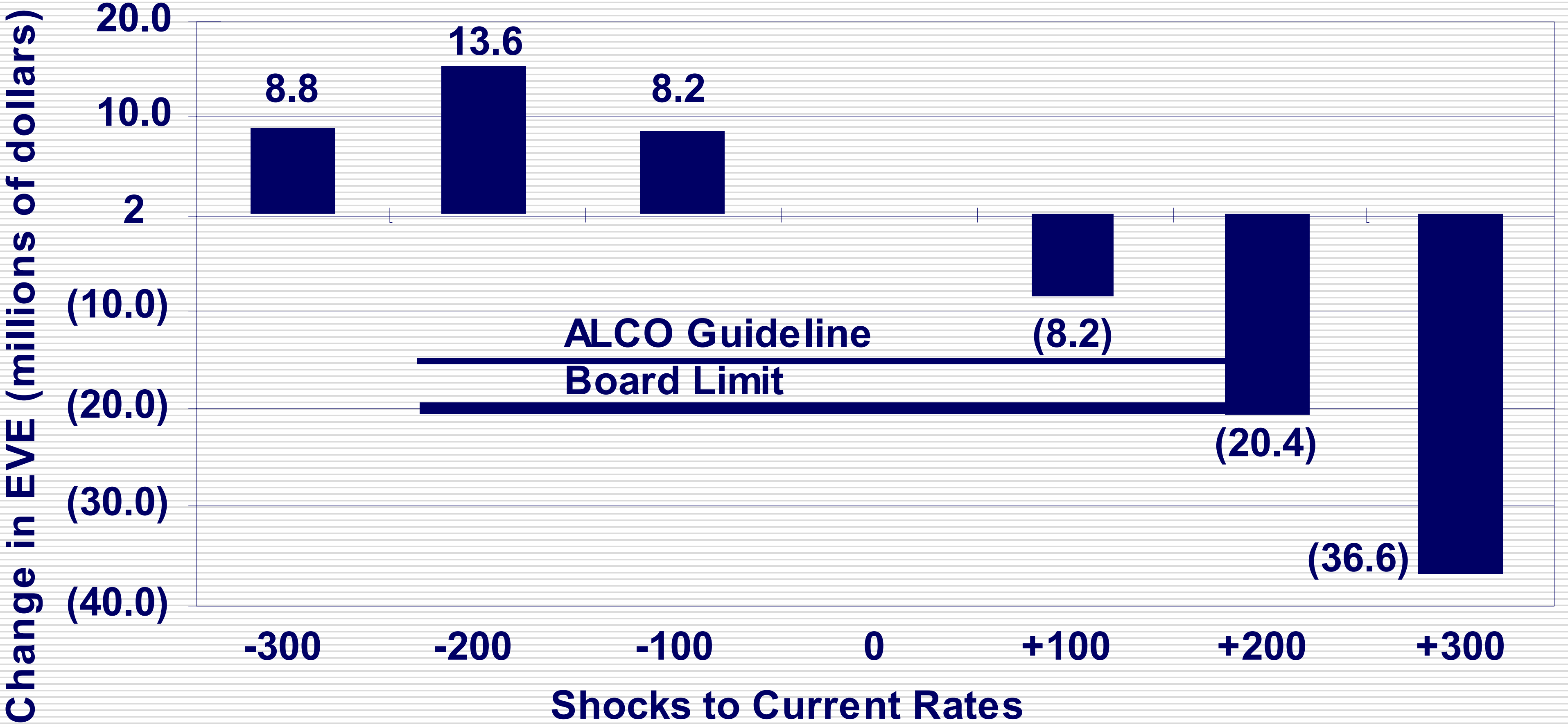
Sensitivity of economic value of equity (MVE) versus most likely (zero shock) interest rate scenario



Effective “duration” of equity

- By definition, duration measures the percentage change in market value for a given change in interest rates
- Hence, a bank’s duration of equity measure the percentage change in MVE that will occur with a 1 percent change in rates:
- Effective duration of equity

  9.9 yrs. = 8,200 / 82,563

Asset / liability sensitivity and DGAP

- Funding GAP and Duration GAP are not directly comparable.
  - Funding GAP examines various “time buckets” while DGAP represents the entire balance sheet.
- Generally, if a bank is liability (asset) sensitive in the sense that net interest income falls (rises) when rates rise and vice versa, it will likely have a positive (negative) DGAP suggesting that assets are more price sensitive than liabilities, on average.

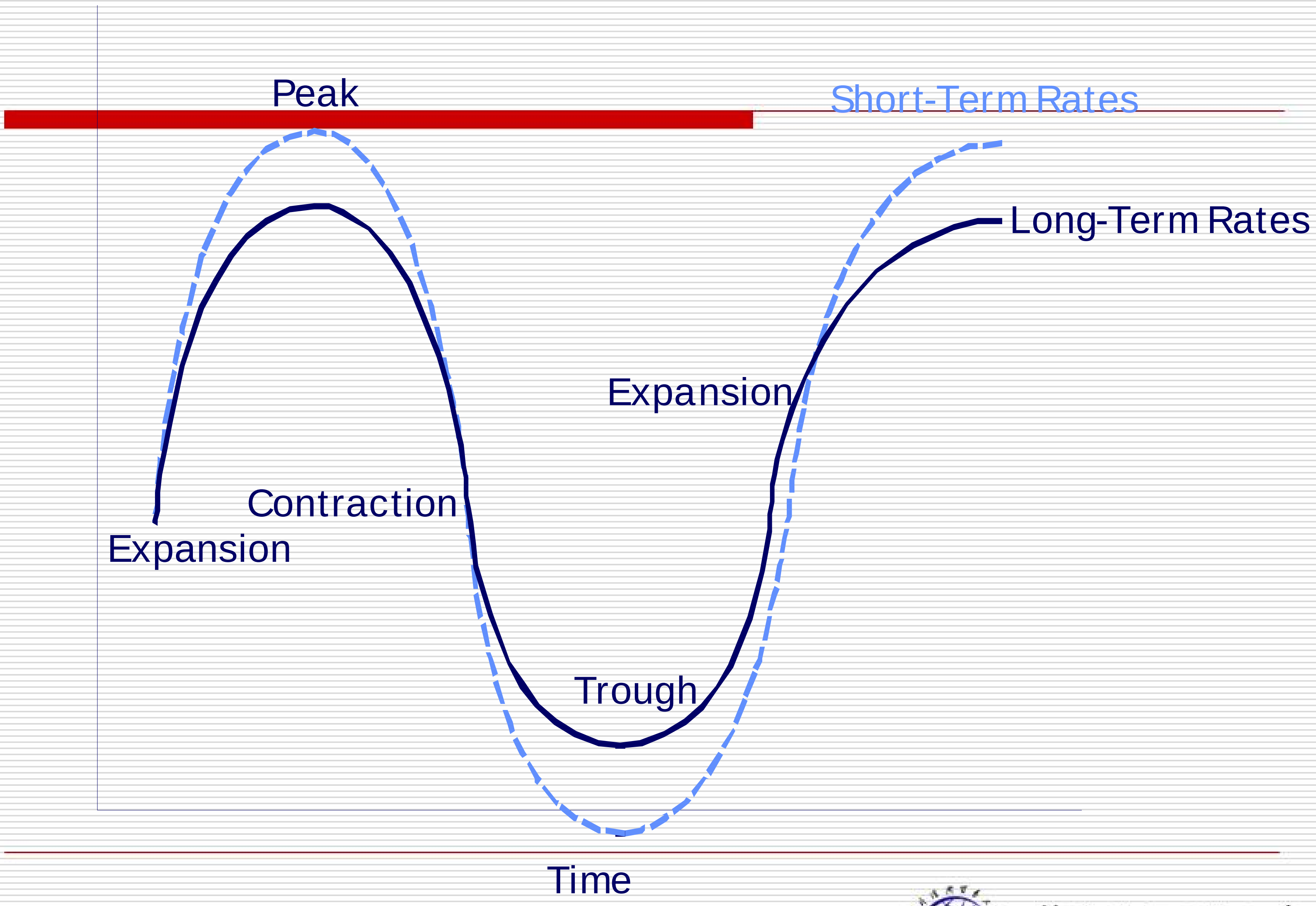
Advantages of DGAP over Funding GAP

- DGAP analysis has the advantage of focusing on all cash flows from the underlying assets and liabilities and not just cash flows that are expected to arise over short time intervals.
- Interest rate risk can be summarized in one measure for the entire portfolio.

Speculating on DGAP

- It is difficult to consistently alter either GAP or DGAP on the balance sheet and increase earnings or the market value of stockholders' equity.
- Whenever management chooses to change asset and liability maturities and/or durations in anticipation of rate changes, it is placing a bet against forward rates from the yield curve.

Interest Rates and the Business Cycle



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